A STUDY OF FINAL STATE EFFECTS
IN THE ELECTRODISINTEGRATION OF A POLARIZED
HELIUM-3 TARGET

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An approach for the description of the final state interaction in the evaluation of inclusive electromagnetic responses of a polarized ^{3}He target, is briefly illustrated. Preliminary results of calculations, where the final state interaction is fully taken into account for the two-body break-up channel, are compared with experimental data, showing a very encouraging improvement with respect to the plane wave impulse approximation results.

1. Introduction

The relevance of polarized ^{3}He targets for investigating the electromagnetic (em) properties of the neutron is well known, and a huge amount of experimental and theoretical work has been devoted to this issue (see, e.g., Refs. 1, 2 and 3 for an overview of the experimental status). The main motivations for such a viable activity is the absence in nature of free-neutron targets and the attempt of devising an effective neutron target, by means of a polarized ^{3}He target. From the theoretical side, the difficulties come from the non trivial task of disentangling the neutron information from the nuclear-structure effects. In particular many effects may play a relevant role, like: i) the "small" components of the bound-state wave function; ii)

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the Δ excitation; iii) the inclusion of the final state interaction (FSI), between the knocked-out nucleon and the interacting spectator pair; iv) the meson exchange currents (MEC); v) relativity. As a first step, the analysis of inclusive em responses of polarized ^{3}He, in the region of the quasi-elastic (qe) peak, was carried out within the *plane wave impulse approximation* (PWIA)[4,5,6,7], where realistic nucleon-nucleon interactions and a relativistic electron-nucleon cross section were adopted. In the final state, only the interaction between the interacting spectator pair and the knocked-out nucleon was disregarded.

Recently, a step forward was performed through calculations of the em responses which include FSI, but within a non relativistic framework[8,9].

In this contribution, a preliminary report (see also Ref. 10) of the inclusive em responses calculated by taking into account both relativistic effects and FSI in the two-body break-up channel, will be presented.

2. The polarized cross section in PWIA

The inclusive scattering of polarized electrons (with helicity h) by a polarized ^{3}He target ($\vec{e} + {}^3\vec{\mathrm{He}} \rightarrow e' + X$) is given by

$$\frac{d^2\sigma(h)}{d\Omega d\omega} = \Sigma + h\,\Delta \tag{1}$$

with

$$\Sigma = \sigma_{Mott} \left[\left(\frac{Q^2}{|\vec{q}|^2}\right)^2 R_L(Q^2,\omega) + \left(\frac{Q^2}{2|\vec{q}|^2} + tan^2\frac{\theta_e}{2}\right) R_T(Q^2,\omega)\right] \tag{2}$$

$$\Delta = -\sigma_{Mott}\, tan\frac{\theta_e}{2} \left\{ cos\theta^*\, R_{T'}(Q^2,\omega)\, \frac{(\epsilon_i + \epsilon_f)}{|\vec{q}|}tan\frac{\theta_e}{2} + \right.$$
$$\left. - \frac{Q^2}{|\vec{q}|^2\,\sqrt{2}}\, sin\theta^* cos\phi^*\, R_{TL'}(Q^2,\omega) \right\} \tag{3}$$

where θ_e is the scattering angle, $\epsilon_{i(f)}$ the energy of the initial (final) electron, θ^* and ϕ^* are the azimuthal and polar angles of the target polarization vector, with respect to the direction of the three-momentum transfer $\vec{q}$; $Q^2 = |\vec{q}|^2 - \omega^2$, with ω the energy transfer. The unpolarized (R_L, R_T) and polarized ($R_{T'}$, $R_{TL'}$) inclusive responses contain the nuclear-structure effects. Finally the asymmetry, A, is defined as A$= \Delta/\Sigma$.

In order to fully appreciate the step forward represented by fully including the interaction in the final three-nucleon system, let us remind that in

our PWIA calculations of the inclusive em responses the following approximation for the final three-nucleon system is considered

$$|j, j_z, T, T_z, \pi, \epsilon_{int}, \alpha; \mathbf{P}\rangle \rightarrow \frac{1}{\sqrt{3}} |\mathbf{p}_f, \sigma_f \tau_f\rangle |\alpha, m_{23}, \tau_{23}; \mathbf{P}_{23}\rangle \qquad (4)$$

with $j(j_z)$ the total angular momentum (third component), $T(T_z)$ the total isospin (third component), π the total parity, ϵ_{int} the intrinsic energy of the three-nucleon system, $\alpha \equiv \{j_{23} T_{23} \lambda_{23}, \pi_{23}, \epsilon_{23}\}$, $\mathbf{P} = \mathbf{p}_f + \mathbf{P}_{23}$ the three-momentum of the three-nucleon center of mass (CM), $|\mathbf{p}_f, \sigma_f \tau_f\rangle$ the plane wave describing the knocked-out nucleon. In Eq. (4), the wave function of the fully-interacting spectator pair is given by $|\alpha, m_{23}, \tau_{23}; \mathbf{P}_{23}\rangle$. The terms that properly antisymmetrize the approximated three-nucleon wave function, are dropped out, since only the direct interaction between the virtual photon and the struck nucleon is taken into account.

In Fig. 1, the experimental asymmetry A (proportional to the transverse (polarized) response, $R_{T'}$, at the qe peak) recently measured at JLab, in the region of the qe peak[1], is compared to our PWIA calculations[6], obtained by using the AV18 nucleon-nucleon potential[11]. At low values of Q^2, calculations[8,9] including FSI and MEC, but within a non relativistic approach, are also shown. At high values of Q^2, in the region close to the quasi-elastic peak, the quite reasonable description of the data achieved by our PWIA (that contains relativistic effects) confirms the physical expectation of the minor role played by FSI, when the nucleon rapidly gets out.

3. Three-nucleon scattering states

The fully-interacting, intrinsic wave function for a three-nucleon system in the continuous spectrum, can be decomposed [12] as follows

$$\Phi^{jj_z TT_z \pi} = \Psi_A^{jj_z TT_z \pi} + \Psi_C^{jj_z TT_z \pi} = \sum_{i=3} \left[\psi_A^{jj_z TT_z \pi}(i) + \psi_C^{jj_z TT_z \pi}(i) \right] (5)$$

where $\Psi_A^{jj_z TT_z \pi}$ is the solution of the Schrödinger equation in the asymptotic region, with two well separated clusters, $\Psi_C^{jj_z TT_z \pi}$ describes the system when the three nucleons are close each other. The intrinsic energy is understood. The functions $\psi_A^{jj_z TT_z \pi}(i)$ and $\psi_C^{jj_z TT_z \pi}(i)$ are Faddeev-like amplitudes, corresponding to the three permutations of the intrinsic coordinates $(\equiv \{\mathbf{r_1}, \mathbf{r_2}, \mathbf{r_3}\})$.

The asymptotic component, $\Psi_A^{jj_z TT_z \pi}$, can be recast in a different way, in order to emphasize its physical content. If one considers the case of a
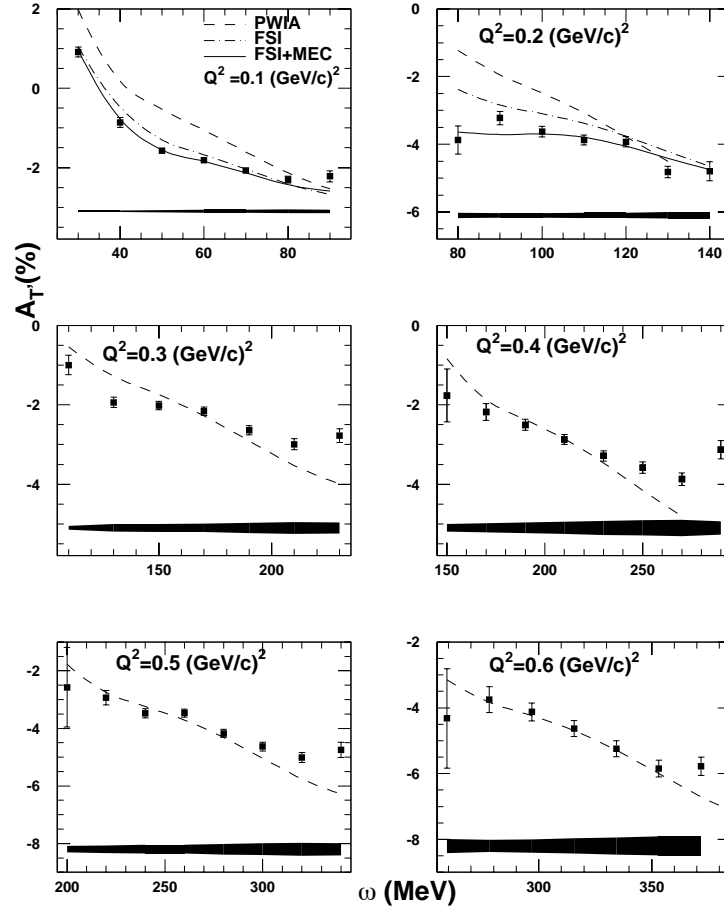
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Figure 1.   The asymmetry A vs the energy transfer, ω, for different values of Q^2. Dashed lines: PWIA calculations within our approach [6]; dash-dotted lines and solid lines: Faddeev calculations with FSI only and with FSI + MEC, respectively[8,9]. Note that A is proportional to the transverese $R_{T'}$ only close to the qe peak. (After W. Xu et al. [1])

N-d scattering state, for the sake of concreteness, one has

$$\Psi_A^{LXjj_z TT_z \pi} = \Omega_{LXj}^R(\mathbf{x}_1, \mathbf{y}_1) +$$
$$+ \sum_{L'X'} {}^j \mathcal{L}_{LL'}^{XX'} \left[\imath \, \Omega_{L'X'j}^R(\mathbf{x}_1, \mathbf{y}_1) + \Omega_{L'X'j}^I(\mathbf{x}_1, \mathbf{y}_1) \right] +$$
$$+ \left[\psi_A^{LXjj_z TT_z}(2) + \psi_A^{LXjj_z TT_z}(3) \right] \tag{6}$$

where L is the relative orbital angular momentum of N with respect to the deuteron, X is the intermediate coupling of the spin of the nucleon with the total angular momentum of the deuteron, and $\{\mathbf{x}_1, \mathbf{y}_1\}$ are the intrinsic coordinates ($\mathbf{x}_1 = \mathbf{r}_2 - \mathbf{r}_3$ and $\mathbf{y}_1 = [\mathbf{r}_2 + \mathbf{r}_3 - 2\mathbf{r}_1]/\sqrt{3}$).

In Eq. (6), $\Omega_{LXj}^{R(I)}$ represents the regular ("irregular", but properly regularized at small distances[12]) solution describing the free scattering of a nucleon by an interacting pair (in this case a deuteron); the matrix $\mathcal{L}$ is given by

$$\mathcal{L} = \frac{\mathcal{S} - 1}{2\imath} = -\pi\mathcal{T} \tag{7}$$

with $\mathcal{S}$ and $\mathcal{T}$ the S-matrix and the T-matrix, respectively. Analogous expressions hold for $\psi_A^{LXjj_zTT_z}(2)$ and $\psi_A^{LXjj_zTT_z}(3)$. In Eq. (6), the first term produces the PWIA, i.e. contains an interacting spectator pair and a free particle; the second term describes the rescattering between the interacting pair and the asymptotically free particle; the third term, $\left[\psi_A^{LXjj_zTT_z}(2) + \psi_A^{LXjj_zTT_z}(3)\right]$, takes care of the correct antisymmetrization of $\Psi_A^{jj_zTT_z\pi}$.

The core component, $\Psi_C^{jj_zTT_z\pi}$, goes to zero for large interparticle distances and energies below the deuteron breakup threshold, while for higher energies, must reproduce a three outgoing particle state. In the approach developed in Ref. 12, $\Psi_C^{jj_zTT_z\pi}$ is explicitly written as an expansion on a basis of Hyperspherical Harmonics Polynomials, with the inclusion of pair-correlation functions, to be determined along with the elements of the S-matrix (see Eq. (7)), through a variational procedure (complex Kohn variational principle). For an illustration of the behavior of the three-nucleon states in coordinate space, see, e.g., Ref. 10.

4. Including FSI

The interaction between the three nucleons in the final state is taken into account in the evaluation of the matrix elements of the em current as follows (note that the current operator is approximated by a sum of one-body operators, without two-body contributions)

$$\langle j', j'_z; T', T'_z, \pi', \epsilon'_{int}; \beta'; \mathbf{q}|J_{IA}^{\mu}(0)|\frac{1}{2}, j_z; \frac{1}{2}, T_z, \pi_b, \epsilon_b; \mathbf{0}\rangle =$$

$$= 3 \sum_{\sigma_1} \sum_{\sigma'_1} \sum_{\sigma_2} \sum_{\sigma_3} \int d\mathbf{k}_1 \ d\mathbf{k}_2 \ \langle j', j'_z; T', T'_z, \pi', \epsilon'_{int}; \beta|\mathbf{k}'_1, \mathbf{k}'_2, \sigma'_1, \sigma_2, \sigma_3\rangle \ \times$$

$$\langle \mathbf{q} + \mathbf{k}_1, \sigma'_1|J_{1,free}^{\mu}(0)|\mathbf{k}_1, \sigma_1\rangle \ \langle \mathbf{k}_1, \mathbf{k}_2, \sigma_1, \sigma_2, \sigma_3|\frac{1}{2}, j_z; \frac{1}{2}, T_z, \pi_b, \epsilon_b\rangle \tag{8}$$

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where $\langle \mathbf{k}_1, \mathbf{k}_2, \sigma_1, \sigma_2, \sigma_3 | j, j_z; T, T_z, \pi, \epsilon \rangle$ indicates the intrinsic three-body wave function, related to the state containing the center of mass motion by the following simplified expression

$$\langle \mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3, \sigma_1, \sigma_2, \sigma_3 | j, j_z; T, T_z, \pi, \epsilon; \mathbf{P} \rangle = \delta(\mathbf{p}_1 + \mathbf{p}_2 + \mathbf{p}_3 - \mathbf{P}) \times$$
$$\langle \mathbf{k}_1, \mathbf{k}_2, \sigma_1, \sigma_2, \sigma_3 | j, j_z; T, T_z, \pi, \epsilon \rangle \tag{9}$$

where $\mathbf{k}_i$ is the spatial part of the four-momentum $k_i^\mu \equiv \{\sqrt{m^2 + |\mathbf{k}_i|^2}, \mathbf{k}_i\} = B_\nu^\mu(\mathbf{P}/M)p_i^\mu$, with B_ν^μ the boost transformation. The expression in Eq. (9) is an approximate one, since the Wigner functions corresponding to the boost transformations, as well as other kinematical factors, are dropped out in this introductory presentation of our approach.
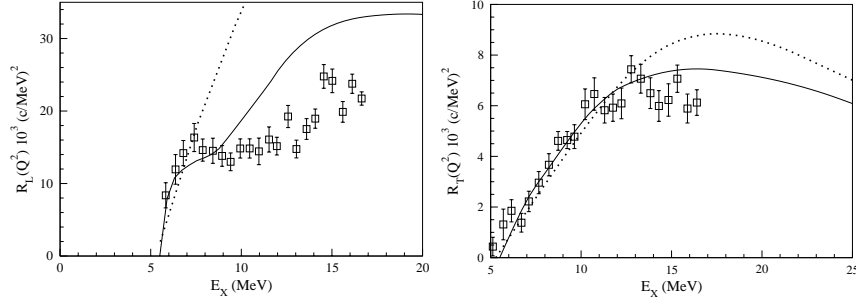

Figure 2.   Unpolarized response functions, R_L and R_T, vs the missing energy $E_X = \sqrt{(\omega + M_{He})^2 - |\vec{q}|^2} - M_{He} \simeq B_3 + \epsilon'_{int}$ for $|\vec{q}| \sim 175$ MeV. Solid line: preliminary results with FSI; dotted line: PWIA. Experimental data from Ref. 13.

Our preliminary calculations based on the matrix elements of the em current of ^{3}He, Eq. (8), both for the unpolarized (R_L and R_T) responses and the polarized asymmetry are presented in Figs. 2,3 and 4. FSI is taken into account only in the two-body break-up channel (where a fully interacting three-nucleon state behaves asymptotically like a pd system). In particular we have included FSI up to $j' \leq 5/2$ for a CM energy of the three-nucleon system $E_{CM}^{fin} \leq 10$ MeV, up to $j' \leq 7/2$ for 10 $MeV < E_{CM}^{fin} \leq 30$ MeV, and up to $j' \leq 11/2$ for 30 $MeV < E_{CM}^{fin} < 100 MeV$. For all other waves we used the PWIA (up to $j' \leq 17/2$ for $Q^2 = 0.1$ $(GeV/c)^2$ and up to $j' \leq 19/2$ for $Q^2 = 0.2$ $(GeV/c)^2$). In the three-body breakup channel we always adopted PWIA. The AV18 nucleon-nucleon interaction [11] has been used, without Coulomb effects nor three-body forces (that will
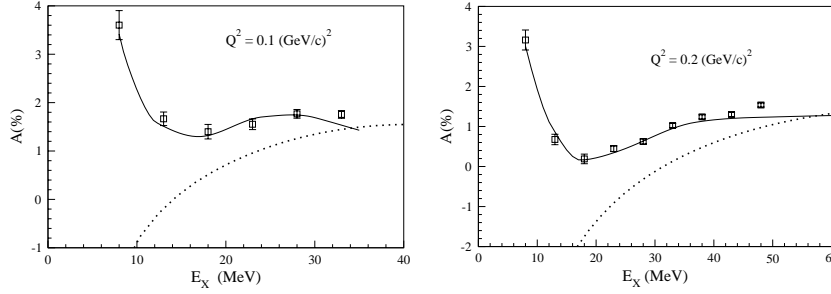
Figure 3. The asymmetry A for $Q^2 = 0.1$ $(GeV/c)^2$ (left) and $Q^2 = 0.2$ $(GeV/c)^2$ (right) vs the missing energy, E_X, at low energy transfer, as in Fig. 2. Solid line: preliminary results with FSI; dotted line: PWIA. Experimental data from F. Xiong et al. [14].

be included in the future). The strong effect of FSI is fully confirmed and we obtain results in qualitative agreement with the ones by the Bochum group[8,9], though in our calculations FSI are presently taken into account only in the two-body break-up channel.
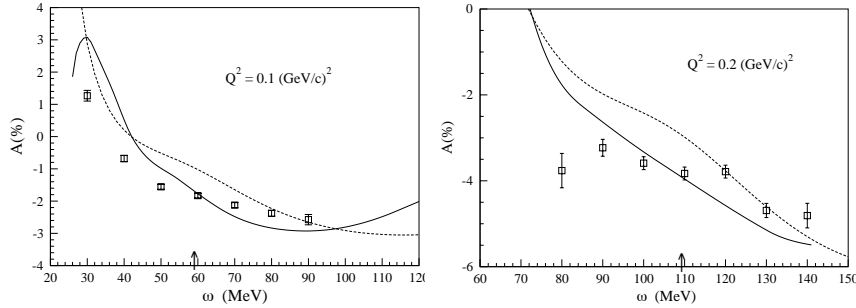

Figure 4. The asymmetry A for $Q^2 = 0.1$ $(GeV/c)^2$ (left) and $Q^2 = 0.2$ $(GeV/c)^2$ (right) vs the energy transfer, ω, in the region of the quasi-elastic peak. Solid line: preliminary results with FSI; dotted line: PWIA. Experimental data from W. Xu et al. [1,2], and private communication. Arrows indicate the qe peak, where the asymmetry A is proportional to $R_{T'}$

5. Summary and Perspectives

Recently the inclusive scattering of polarized electrons by a polarized ^{3}He target have been measured at JLab, both in the region of the quasi elastic peak and in the low-ω wing[1,2]. The direct comparison of calculations corre-

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sponding to different theoretical approaches with these experimental results allows one to better understand the model dependence in the extraction of the neutron em properties, like the magnetic form factor G_M^n. Indeed for a satisfactory interpretation of the data one needs accurate theoretical calculations that include effects beyond the PWIA, such as i) FSI, ii) MEC, iii) relativistic effects and iv) contributions from the explicit presence of the Δ excitation in the ground state of ^{3}He. In our calculations we have adopted both relativistic kinematics and a relativistic electron-nucleon cross section, and we have taken into account exactly the FSI in the two-body break-up channel, by using the three-nucleon wave functions obtained by the Pisa group[12] within a variational approach for both the bound and the excited states.

The development of our approach will follow two distinct paths: i) a better treatment of the relativistic effects within the so called Relativistic Hamiltonian Dynamics (see, e.g., B.D. Keister and W. Polyzou [15]), that allows a Poincaré covariant description of an interacting system with a fixed number of particles; ii) the inclusion of FSI in the three-body break-up channels, together with Coulomb effects and three-body forces.

Our present technology, based on the overlaps appearing in Eq. (8) can be easily generalized to calculate the pion electroproduction from a polarized ^{3}He target.

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